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ACI 318-08 Slender Column Requirements in Sway Frames

1. Design information

Units: $kips := 1000lbf$

Geometry:

Width along X: $b := 18\cdot in$ Width along Y: $h := 18\cdot in$ $A_g := b\cdot h$ $A_g = 324\cdot in^2$

Reinforcement: 4 - #10, All Sides Equal $I_{se} := 214.0in^4$

$I_x := \frac{b\cdot h^3}{12}$ $I_x = 8748\cdot in^4$ $r := \sqrt{\frac{I_x}{A_g}}$ $r = 5.1962\cdot in$

Clear height: $l_u := 16.00\cdot ft$ $l_u = 192\cdot in$

Concrete:

$f'_c := 5\cdot ksi$ $w_c := 150\frac{lbf}{ft^3}$ $E_c := 57000\cdot\sqrt{\frac{f'_c}{psi}}\cdot psi$ $E_c = 4031\cdot ksi$ (ACI 318. 8.5.1)

Reinforcing Steel:

$f_y := 60ksi$ $E_s := 29000ksi$ $0.2\cdot E_c\cdot I_x + E_s\cdot I_{se} = 1.326\times 10^7\cdot kip\cdot in^2$

2. Loading Information

Load Type	Load Factors	Axial Load	Story Shear	Bending Moment (Top & Bottom)	Sustained Load Ratio
Dead Load, D	$f_D := 1.2$	$P_D := 380.0\cdot kips$	$\Sigma V_D := 0\cdot kips$	$M_{D.top} := 32.0\cdot ft\cdot kips$ $M_{D.bot} := 54.0\cdot ft\cdot kips$	$St_D := .635$
Live Load, L	$f_L := 0.5$	$P_L := 140.0\cdot kips$	$\Sigma V_L := 0\cdot kips$	$M_{L.top} := 20.0\cdot ft\cdot kips$ $M_{L.bot} := 36.0\cdot ft\cdot kips$	$St_L := 0$
Snow Load, S	$f_S := 0.0$	$P_S := 0.0\cdot kips$	$\Sigma V_S := 0\cdot kips$	$M_{S.top} := 0\cdot ft\cdot kips$ $M_{S.bot} := 0\cdot ft\cdot kips$	$St_S := 0$
Wind Load, W	$f_W := 1.6$	$P_W := 0.0\cdot kips$	$\Sigma V_W := 0\cdot kips$	$M_{W.top} := 50.0\cdot ft\cdot kips$ $M_{W.bot} := 50.0\cdot ft\cdot kips$	$St_W := 0$
Seismic Load, EQ	$f_{EQ} := 0.0$	$P_{EQ} := 0\cdot kips$	$\Sigma V_{EQ} := 0\cdot kips$	$M_{EQ.top} := 0\cdot ft\cdot kips$ $M_{EQ.bot} := 0\cdot ft\cdot kips$	$St_{EQ} := 0$

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Column loads:

$P_u := f_D \cdot P_D + f_L \cdot P_L + f_S \cdot P_S + f_W \cdot P_W + f_{EQ} \cdot P_{EQ}$ $\quad P_u = 526 \cdot kips$

$P_{u.st} := f_D \cdot P_D St_D + f_L \cdot P_L St_L + f_S \cdot P_S St_S + f_W \cdot P_W \cdot St_W + f_{EQ} \cdot P_{EQ} \cdot St_{EQ}$ $\quad P_{u.st} = 289.56 \cdot kips$

Factored end moments due to loads that cause no appreciable sidesway

$M_{ns.top} := f_D \cdot M_{D.top} + f_L \cdot M_{L.top} + f_S \cdot M_{S.top}$ $\quad M_{ns.top} = 48.4 \cdot ft \cdot kips$

$M_{ns.bot} := f_D \cdot M_{D.bot} + f_L \cdot M_{L.bot} + f_S \cdot M_{S.bot}$ $\quad M_{ns.bot} = 82.8 \cdot ft \cdot kips$

Factored end moments due to loads that cause appreciable sidesway

$M_{s.top} := f_W \cdot M_{W.top} + f_{EQ} \cdot M_{EQ.top}$ $\quad M_{s.top} = 80 \cdot ft \cdot kips$

$M_{s.bot} := f_W \cdot M_{W.bot} + f_{EQ} \cdot M_{EQ.bot}$ $\quad M_{s.bot} = 80 \cdot ft \cdot kips$

Factored total end moments

$M_{u.top} := M_{ns.top} + M_{s.top}$ $\quad M_{u.top} = 128.4 \cdot ft \cdot kips$

$M_{u.bot} := M_{ns.bot} + M_{s.bot}$ $\quad M_{u.bot} = 162.8 \cdot ft \cdot kips$

Story loads for all columns:

The total factored vertical story load: $\Sigma P_u := 27.333 \cdot P_u$ $\quad \frac{\Sigma P_u}{P_u} = 27.333$ $\quad \Sigma P_u = 14377.16 \cdot kips$

$\Sigma V_u := f_D \cdot \Sigma V_D + f_L \cdot \Sigma V_L + f_S \cdot \Sigma V_S + f_W \cdot \Sigma V_W + f_{EQ} \cdot \Sigma V_{EQ}$

$\Sigma V_{u.st} := f_D \cdot \Sigma V_D St_D + f_L \cdot \Sigma V_L St_L + f_S \cdot \Sigma V_S St_S + f_W \cdot \Sigma V_W \cdot St_W + f_{EQ} \cdot \Sigma V_{EQ} \cdot St_{EQ}$

$\Sigma V_u = 0 \cdot kips$ $\quad \Sigma V_{u.st} = 0 \cdot kips$

3. Determine column designation as sway/nonsway and slenderness considerations

Columns and stories within a structure are designated sway or nonsway in accordance with the stability index for a story, Q, per ACI 318 10.10.5. Consider a column in a sway frame/story.

Determine k_{ns} (nonsway frames) and k_s (sway frames) from the alignment charts [ACI 318 Fig. 10.10.1.1. (a) and (b)] respectively or alternate simplified equations used in spColumn Software.

$k_{ns} := 0.800$ $\quad k_s := 1.370$ $\quad \frac{k_s \cdot l_u}{r} = 50.622$

Slenderness effects must be considered in sway frames if $k_s l_u$ / r > 22 (ACI 318-08 10.10.1.a)

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4. Moment magnification at ends of compression member (Sway) ACI 318-08 10.10.7

Magnify column moments per 10.10.7 for second order effects at ends of member as required by 10.10.5

$\beta_{ds} := \dfrac{\Sigma V_{u.st}}{\Sigma V_u}$ $\qquad \beta_{ds} = 0.000$ $\qquad$ (ACI Cl. 10.10.4.2)

$EI_s := \left[\dfrac{(0.2 \cdot E_c \cdot I_x + E_s \cdot I_{se})}{(1 + \beta_{ds})}\right]$ $\qquad EI_s = 1.33 \times 10^7 \cdot kip \cdot in^2$ $\qquad$ (ACI Eq. 10-14)

Critical Load: $P_{c.s} := \dfrac{\pi^2 \cdot EI_s}{(k_s \cdot l_u)^2}$ $\qquad P_{c.s} = 1891.15 \cdot kip$ $\qquad$ (ACI Eq. 10-13)

The sum of critical load in all columns in the story considered: $\Sigma P_{c.s} := 28.649 \cdot P_{c.s}$ $\qquad \dfrac{\Sigma P_{c.s}}{P_{c.s}} = 28.649$ $\qquad \Sigma P_{c.s} = 54179.67 \cdot kips$

$\delta_s := \dfrac{1.0}{1 - \dfrac{\Sigma P_u}{0.75 \cdot \Sigma P_{c.s}}}$ $\qquad \delta_s = 1.548$ $\qquad$ (ACI Eq. 10-21)

Magnify column moments M_1 and M_2 due to second order effects at the ends:

$M_{u.top.second} := M_{ns.top} + \delta_s \cdot M_{s.top}$ $\qquad M_{u.top.second} = 172.203 \cdot ft \cdot kips$

$M_{u.bot.second} := M_{ns.bot} + \delta_s \cdot M_{s.bot}$ $\qquad M_{u.bot.second} = 206.603 \cdot ft \cdot kips$

Set M2 to the larger of the absolute value of top and bot second order magnified moments.

$M_{2.second} := \max\left(\left|M_{u.top.second}\right|, \left|M_{u.bot.second}\right|\right)$ $\qquad M_{2.second} = 206.603 \cdot ft \cdot kips$

$M_{1.second} := \min\left(\left|M_{u.top.second}\right|, \left|M_{u.bot.second}\right|\right)$ $\qquad M_{1.second} = 172.203 \cdot ft \cdot kips$

Determine the first order factored moment, $M_{2.first}$ at the end at which the larger second order end moment, M_2.second acts.

$M_{2.first} := \text{if}\left(\left|M_{u.top.second}\right| \ge \left|M_{u.bot.second}\right|, \left|M_{u.top}\right|, \left|M_{u.bot}\right|\right)$ $\qquad M_{2.first} = 162.8 \cdot ft \cdot kips$

$M_{1.first} := \text{if}\left(\left|M_{u.top.second}\right| \ge \left|M_{u.bot.second}\right|, \left|M_{u.bot}\right|, \left|M_{u.top}\right|\right)$ $\qquad M_{1.first} = 128.4 \cdot ft \cdot kips$

The ratio of second order moments to first order moments after moment magnification at column ends (sway effect) is:

$\dfrac{M_{2.second}}{M_{2.first}} = 1.269$ $\qquad \dfrac{M_{1.second}}{M_{1.first}} = 1.341$

The next step in this calculation was not required in earlier codes unless the parameter $k"l_u/r$ exceeded 35 as previously given in 318-05 10.13.5. This condition amplified the design moment but no stability check was required except for 318-05 10.13.6.

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5. Moment magnification along length of compression member (nonsway) ACI 318-08 10.10.6

Continue to magnify column moments per 10.10.6 for second order effects along the length of member as required by 10.10.2.2.

Determine stiffness reduction factor for sustained axial load

$\beta_{dns} := \min\left(\frac{P_{u.st}}{P_u}, 1.0\right)$ $\beta_{dns} = 0.550$ (ACI 10.10.6.2)

$EI_{ns} := \left[\frac{(0.2 \cdot E_c \cdot I_x + E_s \cdot I_{se})}{(1 + \beta_{dns})}\right]$ $EI_{ns} = 8.55 \times 10^6 \cdot kip \cdot in^2$ (ACI Eq. 10-14)

$P_{c.ns} := \frac{\pi^2 \cdot EI_{ns}}{(k_{ns} \cdot l_u)^2}$ $P_{c.ns} = 3576.99 \cdot kips$ (ACI Eq. 10-13)

Determine and select whether the column is bent in single (1) or double (2) curvature due to second order moments. $Curve := 1$

If column is bent in single curvature, M_1, the smaller of top and bot second order magnified moment is to be taken as positive. Otherwise, M_1 is to be taken as negative.

$M_{1.second} := \text{if}(Curve = 1, M_{1.second}, -M_{1.second})$ $M_{1.second} = 172.203 \text{ ft} \cdot kips$

$C_m := 0.6 + 0.4 \frac{M_{1.second}}{M_{2.second}}$ $C_m = 0.933$ (ACI Eq. 10-16)

Set C_m to 1.0 if M_2, or M_1 and M_2 are equal to zero. $C_m := \text{if}\left(M_{2.second} = 0, 1, 0.6 + 0.4 \cdot \frac{M_{1.second}}{M_{2.second}}\right)$ $C_m = 0.933$

$C_m := \max(C_m, 0.4)$ $C_m = 0.933$ $C_m := \min(C_m, 1)$ $C_m = 0.933$

$\delta := \max\left[1, \frac{C_m}{1 - \left(\frac{P_u}{0.75 \cdot P_{c.ns}}\right)}\right]$ $\delta = 1.161$ (ACI Eq. 10-12)

Check factored moment M_2 is not less than required by 10.10.6.5 equation 10-17 for minimum moment:

$M_{2.second} = 206.603 \text{ ft} \cdot kips$ $M_{2.first} = 162.8 \text{ ft} \cdot kips$

$M_{min} := P_u \cdot (0.6in + 0.03 \cdot h)$ $M_{min} = 49.97 \cdot ft \cdot kips$

Determine final moment value amplified per equation 10-11:

$M_2 := \max(M_{2.second}, M_{min})$ $M_2 = 206.6 \cdot ft \cdot kips$

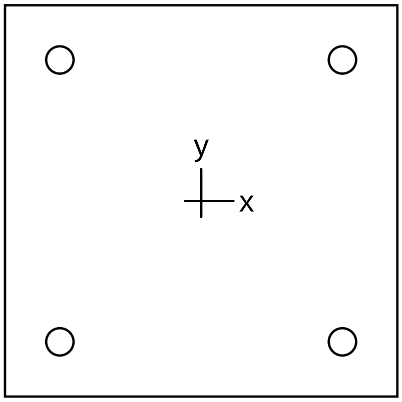
$M_c := \delta \cdot M_2$ $M_c = 239.88 \cdot ft \cdot kips$

Check the ratio of second order moment including further moment magnification effects along the length of compression member (ACI 318-08 10.10.2.2) to first order moments. ACI 318-08 10.10.2.1. requires the ratio not exceed 1.40.

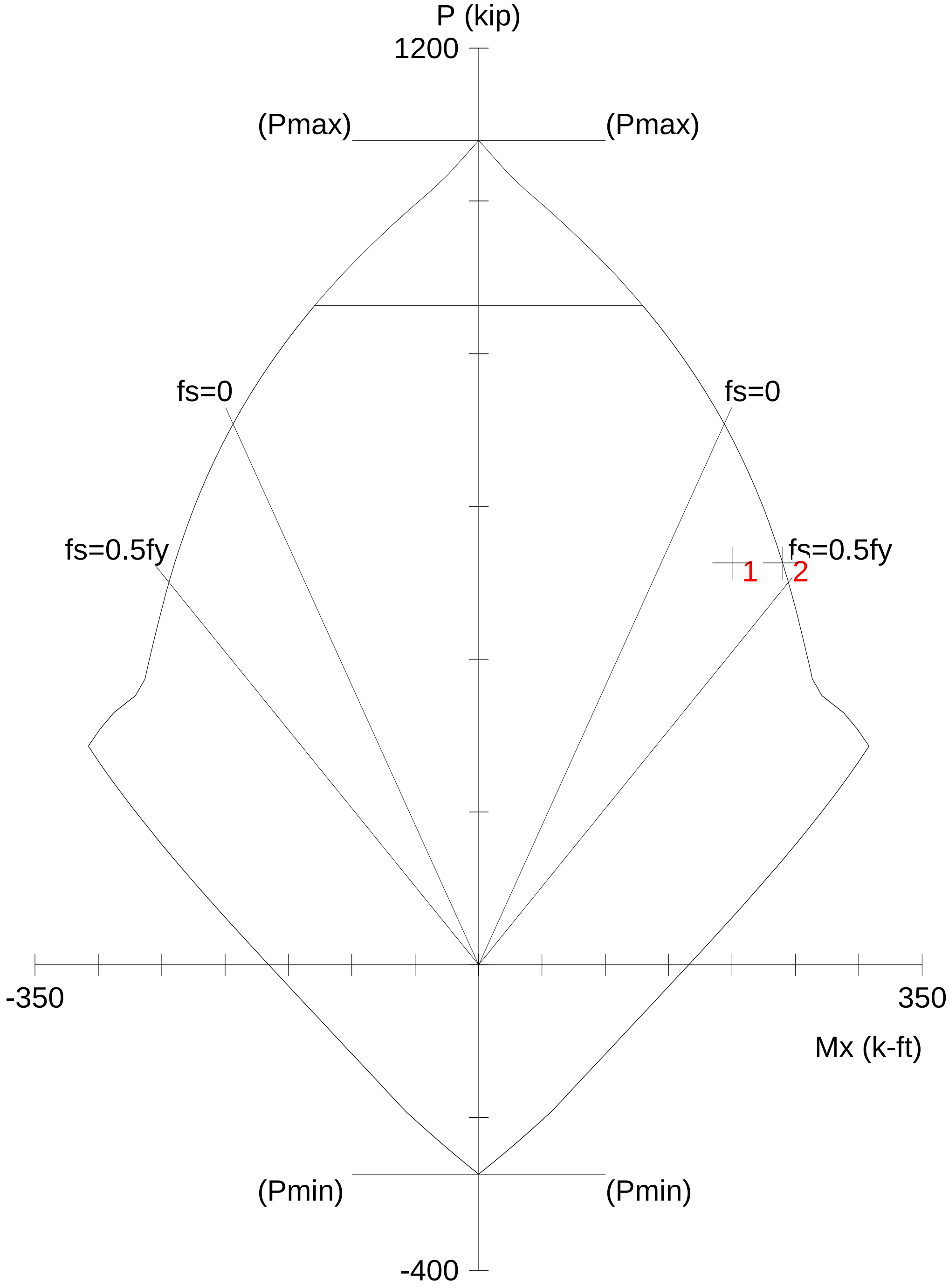
$\frac{M_c}{\max(M_{2.first}, M_{min})} = 1.473$

6. Check column strength based on spColumn output

$\phi P_{n.max} := 863.3 \cdot kips$ $\frac{P_u}{\phi P_{n.max}} = 0.609$ OK $\phi M_{nx} := 239.75 \cdot ft \cdot kips$ $\frac{M_c}{\phi M_{nx}} = 1.001$ NG



18 x 18 in



Code: ACI 318-08

Units: English

Run axis: About X-axis

Run option: Investigation

Slenderness: Considered

Column type: Structural

Bars: ASTM A615

Date: 06/24/10

Time: 13:37:26

File: C:\Users\mdeger\Documents\spColumn v4.60 - ACI 318-08 Cl 10.10.2.2 Implementation\Original MathCad...\CASE B.col

Project: Hassoun 4th Ed Ex 12.4

Column:		Engineer: SP	
f'c = 5 ksi	fy = 60 ksi	Ag = 324 in^2	4 #10 bars
Ec = 4031 ksi	Es = 29000 ksi	As = 5.08 in^2	rho = 1.57%
fc = 4.25 ksi		Xo = 0.00 in	Ix = 8748 in^4
e_u = 0.003 in/in		Yo = 0.00 in	Iy = 8748 in^4
Beta1 = 0.8		Min clear spacing = 11.71 in	Clear cover = 1.88 in
Confinement: Tied		kx(nonsway) = 0.8	kx(sway) = 1.37
phi(a) = 0.8, phi(b) = 0.9, phi(c) = 0.65			

General Information:

File Name: C:\Users\mdeger\Documents\spColumn v4.60 - ACI 318-08 Cl 10.10.2.2 Implemen...\CASE B.col
Project: Hassoun 4th Ed Ex 12.4
Column: Engineer: SP
Code: ACI 318-08 Units: English

Run Option: Investigation Slenderness: Considered
Run Axis: X-axis Column Type: Structural

Material Properties:

f'c = 5 ksi fy = 60 ksi
Ec = 4030.51 ksi Es = 29000 ksi
Ultimate strain = 0.003 in/in
Beta1 = 0.8

Section:

Rectangular: Width = 18 in Depth = 18 in

Gross section area, Ag = 324 in^2
Ix = 8748 in^4 Iy = 8748 in^4
rx = 5.19615 in ry = 5.19615 in
Xo = 0 in Yo = 0 in

Reinforcement:

Bar Set: ASTM A615

Size	Diam (in)	Area (in^2)	Size	Diam (in)	Area (in^2)	Size	Diam (in)	Area (in^2)
# 3	0.38	0.11	# 4	0.50	0.20	# 5	0.63	0.31
# 6	0.75	0.44	# 7	0.88	0.60	# 8	1.00	0.79
# 9	1.13	1.00	# 10	1.27	1.27	# 11	1.41	1.56
# 14	1.69	2.25	# 18	2.26	4.00			

Confinement: Tied; #3 ties with #10 bars, #4 with larger bars.
phi(a) = 0.8, phi(b) = 0.9, phi(c) = 0.65

Layout: Rectangular
Pattern: All Sides Equal (Cover to transverse reinforcement)
Total steel area: As = 5.08 in^2 at rho = 1.57%
Minimum clear spacing = 11.71 in

4 #10 Cover = 1.5 in

Service Loads:

No.	Load Case	Axial Load kip	Mx @ Top k-ft	Mx @ Bot k-ft	My @ Top k-ft	My @ Bot k-ft
1	Dead	380.00	32.00	-54.00	0.00	0.00
	Live	140.00	20.00	-36.00	0.00	0.00
	Wind	0.00	50.00	-50.00	0.00	0.00
	EQ	0.00	0.00	0.00	0.00	0.00
	Snow	0.00	0.00	0.00	0.00	0.00

Sustained Load Factors:

Load Case	Factor (%)
Dead	64
Live	0
Wind	0
EQ	0
Snow	0

Load Combinations:

U1 = 1.200*Dead + 0.500*Live + 1.600*Wind + 0.000*EarthQuake + 0.000*Snow

Slenderness:

Sway Criteria:

X-axis: Sway column. SumPc = 28.65 * Pc SumPu = 27.33 * Pu
Second-order effects along length considered

Column	Axis	Height ft	Width in	Depth in	I in^4	f'c ksi	Ec ksi
Design	X	16	18	18	8748	5	4030.51
Above	X	(no column specified...)					
Below	X	(no column specified...)					

X-Beams Location	Length ft	Width in	Depth in	I in^4	f'c ksi	Ec ksi
Above Left	(no beam specified...)					
Above Right	(no beam specified...)					
Below Left	(no beam specified...)					
Below Right	(no beam specified...)					

Effective Length Factors:

Axis	Psi(top)	Psi(bot)	k(Nonsway)	k(Sway)	klu/r
X	0.000	0.000	0.800	1.370	50.62

Moment Magnification Factors:

Stiffness reduction factor, phi(K) = 0.75
Cracked-section coefficients: cI(beams) = 0.35; cI(columns) = 0.7

0.2*Ec*Ig + Es*Ise (X-axis) = 1.33e+007 kip-in^2

X-axis	At Ends					Along Length					
Ld/Comb	SumPu(kip)	Pc(kip)	SumPc(kip)	Betads	Deltas	Pu(kip)	k'lu/r	Pc(kip)	Betad	Cm	Delta
1 U1	14377.16	1891.03	54176.14	0.000	1.548	526.00	(N/A)	3576.76	0.550	0.933	1.161

Factored Moments due to First-Order and Second-Order Effects:

Minimum eccentricity, Ex,min = 1.14 in

NOTE: Each loading combination includes the following cases:
First line - at column top
Second line - at column bottom

X-axis Load Combo	1st Order Mns k-ft	Ms k-ft	Mu k-ft	Mmin k-ft		2nd Order Mi k-ft	Mc k-ft	Ratio 2nd/1st
1 U1	48.40	80.00	128.40	49.97	M1=	172.21	199.94	1.557 *
	82.80	80.00	162.80	49.97	M2=	206.61	239.88	1.473 *

* Magnified (second-order) moment exceeds 1.4 times first-order moment. Revise column!

Factored Loads and Moments with Corresponding Capacities:

NOTE: Each loading combination includes the following cases:
First line - at column top
Second line - at column bottom

No.	Load Combo	Pu kip	Mux k-ft	PhiMnx k-ft	PhiMn/Mu	NA depth in	Dt depth in	eps_t	Phi
1	1 U1	526.00	199.94	239.75	1.199	11.97	15.49	0.00088	0.650
2		526.00	239.88	239.75	0.999	11.97	15.49	0.00088	0.650 #

Section capacity exceeded. Revise column!

*** End of output ***

```
General Information:
====================
  File Name: C:\Users\mdeger\Documents\spColumn v4.60 - ACI 318-08 Cl 10....\CASE B-Control Points.col
  Project:  Hassoun 4th Ed Ex 12.4
  Column:                                 Engineer: SP
  Code:     ACI 318-08                    Units: English

  Run Option: Investigation               Slenderness: Not considered
  Run Axis:   X-axis                      Column Type: Structural

Material Properties:
====================
  f'c   = 5 ksi                           fy   = 60 ksi
  Ec    = 4030.51 ksi                     Es   = 29000 ksi
  Ultimate strain = 0.003 in/in
  Beta1 = 0.8

Section:
========
  Rectangular: Width = 18 in              Depth = 18 in

  Gross section area, Ag =  324 in^2
  Ix =  8748 in^4                         Iy =  8748 in^4
  rx =  5.19615 in                        ry =  5.19615 in
  Xo =  0 in                              Yo =  0 in

Reinforcement:
==============
  Bar Set: ASTM A615
  Size Diam (in) Area (in^2)   Size Diam (in) Area (in^2)   Size Diam (in) Area (in^2)
  ---- --------- -----------   ---- --------- -----------   ---- --------- -----------
  #  3      0.38        0.11   #  4      0.50        0.20   #  5      0.63        0.31
  #  6      0.75        0.44   #  7      0.88        0.60   #  8      1.00        0.79
  #  9      1.13        1.00   # 10      1.27        1.27   # 11      1.41        1.56
  # 14      1.69        2.25   # 18      2.26        4.00

  Confinement: Tied; #3 ties with #10 bars,  #4 with larger bars.
  phi(a) = 0.8,  phi(b) = 0.9,  phi(c) = 0.65

  Layout: Rectangular
  Pattern: All Sides Equal  (Cover to transverse reinforcement)
  Total steel area: As = 5.08 in^2 at rho = 1.57%
  Minimum clear spacing = 11.71 in

  4 #10   Cover = 1.5 in

Control Points:
===============
                        Axial Load P     X-Moment     Y-Moment NA depth Dt depth    eps_t    Phi
  Bending about                  kip         k-ft         k-ft       in       in
  --------------------- ------------ ------------ ------------ -------- -------- -------- ------
   X @ Max compression        1079.1        -0.00        -0.00    49.91    15.49 -0.00207  0.650
     @ Allowable comp.         863.3       129.29         0.00    18.76    15.49 -0.00052  0.650
     @ fs = 0.0                708.2       193.76         0.00    15.49    15.49 -0.00000  0.650
     @ fs = 0.5*fy             500.7       244.29         0.00    11.52    15.49  0.00103  0.650
     @ Balanced point          357.7       265.43         0.00     9.17    15.49  0.00207  0.650
     @ Tension control         286.0       308.02         0.00     5.81    15.49  0.00500  0.900
     @ Pure bending             -0.0       165.69         0.00     2.50    15.49  0.01557  0.900
     @ Max tension            -274.3         0.00         0.00     0.00    15.49  9.99999  0.900

  -X @ Max compression        1079.1        -0.00        -0.00    49.91    15.49 -0.00207  0.650
     @ Allowable comp.         863.3      -129.29        -0.00    18.76    15.49 -0.00052  0.650
     @ fs = 0.0                708.2      -193.76        -0.00    15.49    15.49  0.00000  0.650
     @ fs = 0.5*fy             500.7      -244.29         0.00    11.52    15.49  0.00103  0.650
     @ Balanced point          357.7      -265.43         0.00     9.17    15.49  0.00207  0.650
     @ Tension control         286.0      -308.02         0.00     5.81    15.49  0.00500  0.900
     @ Pure bending             -0.0      -165.69        -0.00     2.50    15.49  0.01557  0.900
     @ Max tension            -274.3         0.00         0.00     0.00    15.49  9.99999  0.900

  *** End of output ***
```